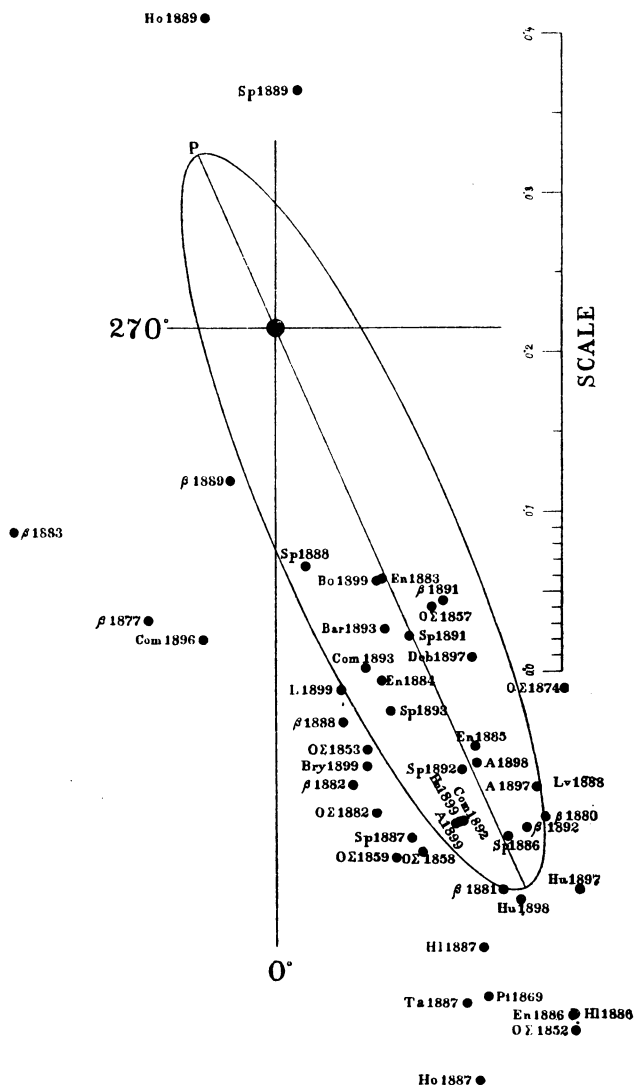

THE ORBIT OF δ *EQUULEI*, OΣ 535.

PUBLICATIONS

OF THE

Astronomical Society of the Pacific.

VOL. XII. SAN FRANCISCO, CALIFORNIA, DECEMBER 1, 1900. NO. 76.

TRIAL ELEMENTS OF THE ORBIT OF δ *EQUULEI*, OΣ 535.

BY WILLIAM J. HUSSEY.

The periodic time of this binary, according to the elements which I have obtained, is 5.7 years. This is only a little more than half of the period of κ *Pegasi*, whose time of revolution according to BURNHAM'S elements is 11.37 years, and which has hitherto been accepted* as the shortest known among the visual double stars.

My elements of δ *Equulei* are to be regarded as tentative — to be accepted if they represent the motion in the future, to be rejected if they fail in this respect. They have been obtained in the course of an investigation which has had for its object the explanation of the very rapid change in the relative positions of the components of this pair during the past fifteen months. In the summer of 1899 this star was measured with comparative ease with the great telescope. The distance was then fully three tenths of a second. Early in September, 1900, Professor AITKEN called my attention to the fact that it had become an exceedingly difficult pair. It is now too close for exact measurement. We have each examined it on a number of nights recently with the 36-inch telescope when the atmospheric conditions were favorable, and have found it either apparently round or so slightly elongated as to render the results of attempts to measure the

*In 1897, SEE published elements of β 883, with a period of 5.5 years. In Volume I of the *Publications of the Yerkes Observatory*, Professor BURNHAM states that from a careful consideration of all the measures of this star, it is practically certain that the period of five and one half years is not correct, and that it is probable that the period will not be far from that of GLASENAPP, which is 16.35 years. LEWIS, using observations from 1879 to 1900, inclusive, obtains elements with a period of 15.8 years.

Professor AITKEN has followed this star very closely during the past five years, measuring it at short intervals to obtain as nearly as possible a continuous series of observations. His results indicate a much longer period than five and one half years.

direction of elongation very uncertain. According to the elements of its orbit heretofore published, the apparent distance should now be somewhat more than a quarter of a second. As a matter of fact, it is less than one tenth of a second.

This close pair was discovered and first measured by OTTO STRUVE in 1852. Soon thereafter it was ascertained that its motion is rapid; but it was not until 1880 that sets of measures began to be obtained with that regularity and continuity which are necessary to furnish the data for a reliable determination of its orbit. From his own observations, OTTO STRUVE thought the periodic time was either about thirteen years or half of this amount. He was inclined to accept the latter as the more probable.

After the publication of the sets of measures by BURNHAM for the years 1880 to 1883, inclusive, WROUBLEWSKY made the first determination of the elements of the orbit, obtaining a period of 11.478 years. His results were published in 1887. In 1895, when the available data for such an investigation had increased considerably, SEE derived elements which in their essential features are practically the same as those obtained by WROUBLEWSKY. These were slightly modified when reprinted the next year in his *Evolution of Stellar Systems*. Their results are as follows:—

	WROUBLEWSKY.		SEE (1895).	SEE (1896.)
Period =	11.478 years.		11.45 years.	11.45 years.
T =	1892.03		1892.80	1892.80
Ω =	24°.05	1870	22°.2	22°.2
$\lambda = \pi - \Omega$ =	26 .61	1870	0 .00	0 .0
i =	81 .75	1870	79 .05	79 .0
e =	0 .2011		0 .14	0 .165
a =	0".406		0".452	0".452

The components of δ *Equulei* do not differ much in brightness. In the years 1897, 1898, and 1899, I measured this star thirteen times. On two nights I thought the companion was in the third quadrant, on eight in the first quadrant, and on three I was in doubt. Whenever the atmospheric conditions are unfavorable, it is difficult to determine the quadrant of this pair with certainty. In this respect the published observations present numerous discordances, and in some instances observers appear to have been guided by WROUBLEWSKY'S elements in fixing the quadrant rather than by their

own independent examinations. On account of these discrepancies, it is necessary to alter the position-angles of some of the observations by 180°, in order to arrange the data for a determination of the elements of the orbit.

When the observations are arranged to correspond to a period of nearly eleven and one half years and then plotted, nearly all the points representing them fall into one or the other of two groups, situated upon opposite sides of the origin, in approximately the directions 20° and 200°. On one side the observations are somewhat more numerous than on the other, and the extreme distances there are a little greater. When one attempts to draw an apparent ellipse guided by the consideration that the areas described by the apparent distance vary proportionally to the times of their description, it is found that the observations can be satisfied only by making the apparent ellipse such that the true orbit is nearly circular. It will be noticed that the eccentricity, as given by WROUBLEWSKY and by SEE, is very much smaller than is usually found in double-star orbits.

I computed a system of elements for δ *Equulei* in accordance with the hypothesis that the period is about eleven and one-half years, and at once obtained a smaller value for the eccentricity than those given above. With these provisional elements, I computed the mean anomalies corresponding to all the observed position-angles which appeared to be reliable, my judgment of their reliability being based upon the results obtained in the course of the measurement of the areas described by the apparent distance. When these mean anomalies were plotted, using the time as the independent variable, it was found that they deviated systematically from a straight line, along which they should lie in accordance with theory. Values corresponding to the observations which had been assumed to belong on one side of the origin were situated above the line that best represented them, taken as a whole, while those on the opposite side of the origin fell below it. I examined the effect which a variation of the several elements would have upon this systematic deviation and concluded that to remove it, it would be necessary further to decrease the eccentricity. By successive steps this was done until the eccentricity had become 0.05, or until the true orbit was assumed to be as nearly circular as that of *Jupiter*. Even then the mean anomalies did not conform to theoretical requirements, and it became apparent that this would not be the case

unless the eccentricity was taken equal to zero, or the true orbit assumed to be a circle. Now, for a circular orbit the center of the apparent ellipse coincides with the origin, and consequently an entire group of observations situated in a given quadrant may be transferred bodily to the opposite quadrant, and there be represented by the same ellipse without disturbing the proportionality of the areas described by the apparent distance to the times of their description.

When I made a comparison of the observations with places computed by means of the elements which I had obtained, taking the eccentricity equal to 0.05, I found the agreement of the results satisfactory for those observations which correspond to points in the vicinity of the vertices of the apparent ellipse; that is, for measures whose position-angles do not differ more than ten or fifteen degrees from 20° and 200°. Some of the other observed angles were also well represented, but there was a notable discrepancy between the observed and computed places for the present time, as is also the case with the published orbits quoted above. Since no system of nearly circular elements can account for the rapid change in distance during the past fifteen months, I at once rearranged the observations to correspond to a period of half the length of that which had previously been assumed, and obtained the elements given below. When the mean anomalies corresponding to the observed position-angles were computed by means of these elements and plotted as before, all evidence of systematic variation disappeared. The values, extending over nearly three thousand degrees of mean anomaly, ranged along a straight line with as much precision as could be expected in a quantity which changes so rapidly, 63°.16 a year, and which for the directions given by most of the measures is so largely influenced by possible errors of observation.

My elements are as follows:—

$$\left.\begin{aligned} \text{Period} &= 5.70 \text{ years.} \\ T &= 1901.18 \end{aligned}\right.$$

$$\left.\begin{aligned} \Omega &= 24^\circ.1 \\ \lambda &= 179\ .0 \\ i &= 74\ .5 \end{aligned}\right\} 1900.$$

$$\begin{aligned} e &= 0\ .54 \\ a &= 0''.25 \end{aligned}$$

Motion retrograde.

λ is here reckoned in the direction of increasing position-angles, so that when M, E, and v are counted as usual in the direction of motion, we have the following formulas for computing the position-angle and distance:—

$$\rho \sin(\theta - \Omega) = r \sin(-v + \lambda) \cos i,$$
$$\rho \cos(\theta - \Omega) = r \cos(-v + \lambda),$$

v and r being found by the usual equations.

An ephemeris computed by means of these elements for each ten degrees of mean anomaly, or for each thirty-sixth part of a revolution is given below. It is given in this form in order that it may be available for the comparison of the observations of any revolution. About a third of the time, however, the motion is so rapid as to render a comparison with the ephemeris roughly approximate only.

In the table giving the Comparison of Observed and Computed Places, the Computed Places have been interpolated from the ephemeris when the motion is slow, and computed directly for the date of observation when it is rapid. The observed position-angles have been reduced to the epoch 1900 by applying the corrections for precession, except for those values which are given in integers and which are approximate only.

An inspection of the columns of residuals, O — C, will show the manner in which these elements represent the observations through no less than eight complete revolutions. While some of the residuals are large, the number of this character is perhaps not greater than is to be expected for a star which has been observed so frequently with instruments of such great variety and which at all times is so difficult. In a considerable number of the cases in which the residuals are abnormally large, the observers have particularly noted that the atmospheric conditions were unfavorable, or that the star for other reasons was difficult to measure. Thus, OTTO STRUVE states that the atmospheric conditions were poor at the time of his observation of 1865.91. WINLOCK and SEARLE regarded their measures of 1866.78 as very unsatisfactory.* ENGELMANN'S observations were made with a small telescope, and the difficulty of the star for such an instrument is a sufficient explanation of the magnitude of some of his residuals. HALL'S angle of 1879.77 is published as uncer-

* Professor PICKERING has very kindly examined the original records of these observations for me, and states that but little weight should be given to them.

tain, and LEAVENWORTH states the star was exceedingly difficult with his instrument.

From a casual examination of the column of residuals, it might appear that the late measures of angle by Professor AITKEN are not represented as closely as desirable, and on this account that the several elements require considerable corrections. This may or may not be so. If it is, they should be corrected when the data requisite for the purpose become available. Further observations in other parts of the orbit are desirable to fix with greater certainty the geometrical elements. The periodic time and epoch of periastron passage appear to be well determined. Elsewhere in this number of these *Publications*, Professor AITKEN gives the details of his observations. A reference to his paper will show that he regards his measure of 1900 61 as more reliable than those which he obtained later. I have accordingly given it the preference in the final adjustment of the periodic time and epoch of periastron passage. According to these elements, the angular motion was very rapid during the period covered by these measures, a part of the time amounting to nearly a degree a day, and when we consider the present extreme difficulty of the star, the residuals are more consistent than might have been expected from *a priori* considerations. It may further be remarked in this connection, that all of his measures of angle for the year 1900 will give residuals well within reasonable limits for the possible errors of observation by increasing the date of periastron by only one-twentieth of a year, and if at the same time the periodic time be increased by one one-hundredth of a year, the entire series of residuals from 1852 to 1900 will not be materially affected.

According to my elements, the apparent distance of the companion is greater when it is at periastron (at P in the diagram), than it is before and after that epoch. For a brief period about that time the star should appear distinctly elongated when seen under favorable conditions with a large telescope. Half a year earlier or later it should appear nearly or quite round. From this consideration it appears not improbable that the data may be forthcoming within a few months, which will decide whether the short period given by the elements which I have obtained is correct or not.

LICK OBSERVATORY, November 22, 1900.

COMPARISON OF OBSERVED AND COMPUTED PLACES.

Dates.	Computed.		Observed.		O — C.		No. of Nights	Observers.
	θ	ρ	θ	ρ	$\Delta\,\theta$	$\Delta\,\rho$		
	°	″	°	″	°	″		
1852.66	24.3	0.39	22.8	0.48	— 1.5	+0.09	2	OΣ
1853.91	15.3	0.29	12.3	0.27	— 3.0	—0.02	1	OΣ
1854.69	358.0	0.13	Single.		....	......	1	OΣ
1856.57	43.9	0.16	Single.		....	......	1	OΣ
1857.67	28.7	0.35	29.4	0.20±	+ 0.7	—0.15±	2	OΣ
1858.59	22.9	0.38	15.8	0.34±	— 7.1	—0.04±	1	OΣ
1859.65	14.9	0.29	13.0	0.34±	— 1.9	+0.05±	1	OΣ
1861.51	184.5	0.09	236?	....	....	.....	1	OΣ
1865.91	5.7	0.18	22.7	<0.5	+ 17.0	......	1	OΣ
1866.78	219.2	0.09	230.3	....	+ 11.1	......	1	Wl
1866.78	219.2	0.09	110.3	....	—108.9	......	1	Sr
1869.69	24.7	0.38	5	....	— 20	......	1	Du
1869.75	24.3	0.39	10.7	....	— 13.6	......	2	Wl
1869.75	24.3	0.39	17.8	0.44	— 6.5	+0.05	4, 1	Pi
1870.73	17.9	0.33	8	....	— 10	......	1	Du
1874.67	29.4	0.34	22.1	....	— 7.3	......	1	OΣ
1874.73	28.8	0.35	359.9	....	— 28.9	......	1	OΣ
1874.75	28.8	0.35	38.9	0.29±	+ 10.1	—0.06±	1	OΣ
1877.76	329.9	0.06	336.3	0.2 ±	+ 6.4	+0.14±	1	β
1878.63	181.9	0.08	No certain elong.		....	......	1	β
1878.68	174.0	0.07	180?	....	+ 6	......	1	β
1879.77	35.4	0.25	150?	....	....	......	1	Hl
1880.60	27.8	0.36	29.0	0.35	+ 1.2	—0.01	5	β
1881.46	22.4	0.38	22.0	0.38	— 0.4	0.00	4	β
1882.62	13.0	0.26	11.9	0.31	— 1.1	+0.05	1	OΣ
1882.63	13.0	0.26	9.7	0.29	— 3.3	+0.03	3	β
1883.12	1.8	0.15	23	0.17±	+ 21	+0.02±	4	En
1883.55	303.0	.05	307.5	0.21	+ 4.5	+0.16	3	β
1884.70	73.5	0.07	17	0.23±	— 57	+0.16±	3	En
1885.95	30.3	0.33	25.5	0.29	— 4.8	—0.04	6	En
1886.84	24.4	0.39	23.4	0.47	— 1.0	+0.08	2	Hl
1886.87	24.2	0.39	24.5	0.35	+ 0.3	—0.04	6, 2	Sp
1886.91	23.9	0.39	23.2	0.47	— 0.7	+0.08	4	En
1887.78	18.4	0.34	15.2	0.49	— 3.2	+0.15	2, 1	Ho
1887.79	18.4	0.34	15.8	0.44	— 2.6	+0.10	5	Ta
1887.80	18.1	0.34	18.7	0.41	+ 0.6	+0.07	4	Hl
1887.86	17.6	0.33	15.0	0.33	— 2.6	0.00	11, 8	Sp
1888.60	8.1	0.20	33.4	0.35	+ 25.3	+0.15	4, 2	Lv
1888.69	6.0	0.19	9.9	0.25	+ 3.9	+0.06	4	β
1888.90	357.8	0.13	7.0	0.15	+ 9.2	+0.02	10, 14	Sp
1889.51	227.5	0.08	343.2	0.10	+115.7	+0.02	1	β
1889.82	201.1	0.11	193.1	0.2 ±	— 8.0	+0.09	1	Ho
1889.84	199.7	0.11	175.0	0.15	— 24.7	+0.04	3	Sp

COMPARISON OF OBSERVED AND COMPUTED PLACES—*Continued.*

Dates.	Computed.		Observed.		O — C.		No. of Nights.	Observers.
	θ	ρ	θ	ρ	$\Delta\theta$	$\Delta\rho$		
	°	″	°	″	°	″		
1890.88	40.8	0.18	Single		....	......	3	Sp
1891.63	30.5	0.32	31.6	0.20	+ 1.1	—0.12	5	β
1891.85	28.8	0.35	23.4	0.21	— 5.4	—0.14	5	Sp
1892.39	25.3	0.38	26.6	0.35	+ 1.3	—0.03	4	β
1892.78	22.9	0.38	21.0	0.33	— 1.9	—0.05	2	Com
1892.91	22.2	0.38	22.8	0.30	+ 0.6	—0.08	2	Sp
1893.84	15.1	0.29	14.9	0.22±	— 0.2	—0.07±	2	Com
1893.86	14.8	0.29	20.1	0.20	+ 5 3	—0.09	2	Bar
1893.93	13.9	0.27	16.8	0.25	+ 2.9	—0.02	6	Sp
1894.85	331.2	0.07	Single		....	......	4	Sp
1895.61	194.5	0.10	Single		....	......	1	Com
1895.70	178.2	0.08	Single		....	......	2	L
1896.68	38.6	0.21	346.8	0.20±	— 51.8	—0.01±	1	Com
1897.72	27.6	0.36	30.9	0.24	+ 3.3	—0.12	4, 2	Dob
1897.78	27.3	0.37	28.4	0.40	+ 1.1	+0.03	3	Hu
1897.83	26.5	0.37	29.6	0.33	+ 3.1	—0.04	4	A
1898.49	22.9	0.38	24.8	0.30	+ 1.9	—0.08	4	A
1898.62	22.1	0.38	23.1	0.39	+ 1.0	+0.01	6	Hu
1899.46	15.2	0.30	20.1	0.33	+ 4.9	+0.03	3	A
1899.63	14.0	0.28	21.7	0.17	+ 7.7	—0.11	1	Bo
1899.76	12.4	0.25	10.3	0.23	— 2.1	—0.02	1	L
1899.77	12.3	0.25	11.9	0.28	— 0.4	+0.03	1	Bry
1899.80	10.6	0.23	20.5	0 33	+ 9 9	+0.10	4	Hu
1900.61	319.4	0.06	317.8	....	— 1.6	......	1	A
1900.75	265.7	0.05	287.2	....	+ 21.5	......	1	A
1900.75	265.7	0.05	270.1	....	+ 4.4	......	1	Hu
1900.79	252.8	0.06	275.3?	....	+ 22.5	.....	1	A
1900.86	235.9	0.07	264	...	+ 28	......	1	A

In the above table and in the diagram, the abbreviations used to designate the observers are as follows: OΣ = OTTO STRUVE, Wl = WINLOCK, Sr = SEARLE, Du = DUNÉR, Pi = PIERCE, β = BURNHAM, Hl = HALL, En = ENGELMANN, Sp = SCHIAPARELLI, Ho = HOUGH, Ta = TARRANT, Lv = LEAVENWORTH, Com = COMSTOCK, Bar = BARNARD, L = LEWIS, Dob = DOBERCK, A = AITKEN, Hu = HUSSEY, Bo = BOYER, Bry = BRYANT.

EPHEMERIS.

M.	t—T.	t.	θ	ρ	M.	t—T.	t.	θ	ρ
°	y		°	″	°	y		°	″
0	0.000	1901.180	201.8	0.12	190	3.008	1898.488	22.9	0.38
10	0.158	.338	192.0	0.10	200	3.167	.647	21.9	0.38
20	0.317	.497	170.6	0.07	210	3.325	.805	20.9	0.37
30	0.475	.655	118.5	0.05	220	3.483	.963	19.8	0.36
40	0.633	.813	71.4	0.07	230	3.642	1899.122	18.7	0.34
50	0.792	.972	53.6	0.11	240	3.800	.280	17.5	0.33
60	0.950	1902.130	45.4	0.15	250	3.958	.438	16.0	0.31
70	1.108	.288	40.6	0.19	260	4.117	.597	14.4	0.28
80	1.267	.447	37.4	0.23	270	4.275	.755	12.4	0.25
90	1.425	.605	35.0	0.26	280	4.433	.913	9.9	0.22
100	1.583	.763	33.1	0.29	290	4.592	1900.072	6.5	0.19
110	1.742	.922	31.5	0.31	300	4.750	.230	1.3	0.15
120	1.900	1903.080	30.1	0.33	310	4.908	.388	352.3	0.11
130	2.058	.238	28.9	0.35	320	5.067	.547	332.3	0.07
140	2.217	.397	27.8	0.36	330	5.225	.705	282.3	0.05
150	2.375	.555	26.7	0.37	340	5.383	.963	235.1	0.07
160	2.533	.713	25.8	0.38	350	5.542	1901.022	215.4	0.10
170	2.692	1903.872	24.8	0.38	360	5.700	1901.180	201.8	0.12
180	2.850	1904.030	23.8	0.39					

ON THE PROGRESS MADE IN THE LAST DECADE IN THE DETERMINATION OF STELLAR MOTIONS IN THE LINE OF SIGHT.*

By H. C. Vogel.

After the early attempts at the determination of the component in the line of sight of the motion of the stars by means of the spectroscope, which were made in 1868 by Huggins, in London, and in 1871 by myself at Bothkamp, on a few stars, extensive observations of this kind were conducted at the Observatory at Greenwich, extending over a period of thirteen years. The great persistence exhibited by Maunder in these observations, which were placed in his charge, is worthy of the more recognition in view of the slight interest which astronomers then generally had in the physical side of astronomy, and especially in view of their skeptical attitude toward the application of the

*Reprinted from the *Astrophysical Journal* for June, 1900.